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ABSTRACT

This study examined the effect of various instructional conditions on the ability of young children to form classes and to recognize order and equivalence relations. Children from four kindergarten and four first grade classes were first classified by two levels of IQ and then randomly assigned to experimental and control groups, giving a 2 x 2 x 2 design. The experimental group received 17 lessons on conjunction, disjunction, negation, and selected mathematical relations, designed to help the children learn to form classes, to form intersections, unions, and complements of classes, and to recognize relations between classes and class elements. The posttests consisted of five achievement tests and four transfer tests, and the scores were analyzed by univariate and multivariate analysis of variance. The results showed that the children were able to learn the skills taught, and that some transfer to related skills occurred. The main effects of treatment and IQ were highly significant on all tests, but grade level was not significant. Various implications for early schooling are presented. (MM)

An Investigation in the Learning of Selected Parts of a Boolean Algebra by Young Children

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A paper presented at the Annual 1972 AERA Meeting, Chicago, Illinois.

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Literature pertaining to the theoretical as well as the empirical study of the thinking of young children is quite abundant. The Geneva School has made a significant contribution to both areas. Although used extensively by psychologists, this literature remains largely untapped by mathematics educators with their present preoccupation with behaviorism. However, for mathematics educators interested in cognition, the research literature surrounding the work of the Geneva School provides a framework (1) for explaining how mental operations basic to mathematical thought develop, (2) for identifying structural characteristics of thought as they undergo change with age, and (3) for forming a theoretical basis for certain curricular decisions and experiments in the learning of mathematics.

The present study was designed with the following purposes: (1) To determine if specific instructional conditions improve the ability of young children of various ages and intellectual levels to (a) form classes based on relations between selected attributes and (b) establish the existence of selected equivalence and order relations that exist within and among sets of objects, and (2) To investigate that if specific instructional conditions improve abilities outlined in (a) and (b) of (1) above, whether transfer occurs to (a) other class-related activities and (b) the transitive property of the selected equivalence and order relations.

Grouping Structures

Piaget (1952) has identified four main stages in which structural characteristics of thought are qualitatively different. They are: (1) sensory-motor, preverbal stage; (2) the stage of preoperational representation; (3) the stage of concrete operations; and (4) the stage of formal operations [pp. 9-10].

Concrete operations are a part of the cognitive structure of children from about 7-8 years of age to 11-12 years of age. Piaget (1964) postulates this cognitive structure has the form of what he calls groupings, of which the following five properties exist:

(1) Combinativity: $x + x^1 = y; y + y^1 = z$

(2) Reversibility: $y - x = x^1$ or $y - x^1 = x$

(3) Associativity: $(x + x^1) + y^1 = x + (x^1 + y^1)$

(4) General Operation of Identity: $x - x = 0$

(5) Special Identities: $x + x = x$

Eight major groupings are identified, each of which satisfy the above properties. The idea of an operation is central to these groupings. Piaget (1964) views an operation as being an interiosed action, always linked to other operations, and part of a total structure. For example, an operation could consist of joining objects in a class. Piaget's claim is that fundamental to the understanding of the development of knowledge is the idea of an operation. The groupings are the structures of which the operations are a part. The difference in the groupings reside in the various operations which are structured. The elements of two groupings are classes and asymmetrical relations which correspond to the cognitive operations of combining individuals in classes and assembling

the asymmetrical relations which express differences in the individuals.

It must be made clear that the Geneva School is concerned with describing transformations that intervene between the input of a problem and the output of a solution of the problem by a subject. As Bruner (1959) put it, "Piaget proposes to describe them [the transformations] in terms of their correspondence to formal logical structures [p. 364]." At a certain stage, a child becomes capable of solving a variety of problems not possible at an earlier stage, but still not able to solve other problems which contain elements of a more advanced stage. In short, Piaget has provided a structure of intelligence which can be used to account for success or failure of children when solving certain problems.

Because the grouping structure is used as a tool to characterize the thinking of the young child, it is interesting to give an interpretation.

In <u>The Psychology of Intelligence</u>, Piaget apparently selects special classes for part of his elements in the first grouping. These classes must satisfy the following pattern: $\phi \subset A_1 \subset A_2 \subset \ldots \subset \bigcup A_\sigma$, where $\sigma \varepsilon A$ and A is the index set. If "$\subset$" is interpreted to mean "$\subseteq$", then the above sets constitute a lattice, which is a partially ordered system in which any two elements have a greatest lower bound and a least upper bound. Clearly, "$\subset$" is a partial ordering of the sets in question since it is (1) reflexive, (2) anti-symmetric, and (3) transitive. Moreover, for any two elements A_α and A_β, $A_\alpha \cap A_\beta$ is the greatest lower bound and $A_\alpha \cup A_\beta$ is the least upper bound.

This lattice structure is not all that is included in the first grouping. Classes of the form $A_\sigma' = A_\gamma - A_\sigma$ where $A_\sigma \subset A_\gamma$ are also included. The classes A_σ' included along with the elements of the lattice are the elements of this first grouping. If one interprets Piaget's (1964) "+" to be "$\cup$", then he gives (embedded in a zoological classification) statements analogous to the following [p. 43].:

(1) Combinativity: $A_\sigma \cup A_\sigma' = A_\gamma$

(2) Reversibility: If $A_\sigma \cup A_\sigma' = A_\gamma$, then $A_\sigma = A_\gamma - A_\sigma'$.

(3) Associativity: $(A_\sigma \cup A_\sigma') \cup A_\gamma' = A_\sigma \cup (A_\sigma' \cup A_\gamma')$.

(4) General Opperation of Identity: $A_\sigma \cup \phi = A_\sigma$.

(5) Special Identities: (a) $A_\sigma \cup A_\sigma = A_\sigma$, (b) $A_\sigma \cup A_\gamma = A_\gamma$ where $A_\sigma \subset A_\gamma$.

When considering definitions of a Boolean Algebra such as recorded in Modern Algebra by Birkhoff and MacLane [pp. 336, 37] it can be noted that many aspects of a Boolean Algebra are inherent in Grouping I. Furthermore, the set of all subsets of any set is a Boolean Algebra. Also the algebra of statements under the connectives "and," "or," and "not" is a Boolean Algebra. Indeed, with respect to classes, the conjunction in logic "and" has the same meaning as does intersection in mathematics; the disjunction in logic "or" (in the inclusive sense) has the same meaning as union in mathematics; and the negation in logic "not" has the same meaning as does complementation in mathematics.

Grouping I also describes essential operations and relations involved in cognition of simple hierarchies of classes. Proficiency with the use of the class inclusion relation is viewed by Piaget as essential in the establishment of operatory classification. Two abilities, described by structural properties, are of particular

importance in this proficiency. The first is the ability to compose classes (combinativity) and decompose classes (reversibility), and the second is the ability to hold in mind a total class and its subclasses at the same time, made possible through combinativity and reversibility; or as will be seen later, through an ability to think of two attributes at the same time, made possible by yet another grouping.

Due to the centrality of the class inclusion problem as a test of operatory classification, Piaget (1952) reported of an early study with children of ages four to eight. A major part of the investigation involved presenting the children individually with materials similar to the following: wooden beads, the majority of which were brown; blue beads, the majority of which were square; and flowers, the majority of which were poppies. Typical kinds of questions asked were the following: (1) Are there more wooden beads or more brown beads? (2) Would a necklace made of the wooden or of the brown beads be longer? and (3) Would the bunch of flowers or the bunch of poppies be bigger? The question were very hard for children under seven but children over seven performed quite well. The main reason attributed to the failure of the younger children was that they supposedly could not think simultaneously of the whole and its parts, as mentioned in the paragraph above. A host of studies, both replication studies and training studies, have been conducted around the class inclusion problem, some of which are reported in Sigel and Hooper (1968).

Continuing the "additive" operations, Piaget delineates two groupings entitled "Addition of Asymmetrical Relations" and "Addition

of Symmetrical Relations." The asymmetrical relations referred to are interpreted here as strict partial orderings, i.e., orderings that are (1) transitive, (2) asymmetric, and (3) non-reflexive. Moreover, if such relations are linear, then the set A on which the relation is defined is a chain and hence is a lattice. The general properties of a grouping may be applied. Combinitivity can be interpreted under the more general notion of function composition. That is, $A\alpha B$ and $B\alpha C$ implies $A\alpha C$; $A\alpha B$ and $B\alpha A$ implies $A\alpha A$, etc. The former is transitivity and the latter is denoted as reversibility. The composition is associative by virtue of the transitive property and has special identities (tautology). Addition of symmetrical relations involves several distinct categories of relations; some transitive, some intransitive, some reflexive, and some nonreflexive. The statement of properties are quite similar to the addition of asymmetrical relations.

Piaget (1964) also describes groupings based on multiplicative operations, i.e., those which deal with more than one system of classes or relations at a time. Two of these groupings are called Bi-Univocal Multiplication of Classes and Bi-Univocal Multiplication of Relations. In the former, an example is given by the following: If C_1 and C_2 denote the same set of, say, squares, but $C_1 = A_1 \cup A_2$ and $C_2 = B_1 \cup B_2$ where A_1 denotes red squares, A_2 blue squares, B_1 large squares, and B_2 small squares, then $C = C_1 \cap C_2 = A_1B_1 \cup A_1B_2$ $A_2B_1 \cup A_2B_2$. In other words, a matrix or double entry table of four cells has been generated with the component classes of C_1 on one dimension and those of C_2 on the other. In the case of Bi-Univocal Multiplication of Relations, an example could be seriating a collection

of sticks according to length and thickness. A double entry table would thus be defined. Of course, special restrictions would have to be placed on the objects and relations. If L denotes length and T thickness, then the matrix could look as follows: All the objects in the first row are the same thickness but different lengths while the objects of the first column

$$
\begin{array}{lllll}
L_1T_1 & L_2T_1 & L_3T_1 & L_4T_1 & \cdots \\
L_1T_2 & L_2T_2 & L_3T_2 & L_4T_2 & \cdots \\
L_1T_3 & L_2T_3 & L_3T_3 & L_4T_3 & \cdots \\
L_1T_4 & L_2T_4 & L_3T_4 & L_4T_4 & \cdots \\
\vdots & & & &
\end{array}
$$

are the same length but different thickness. It can be noted that equivalence as well as order relations are involved in this process. The structural properties of these latter two groupings are not discussed--except to say that multiplication of classes allows a child to classify according to two or more classification systems at once--or to consider an object as possessing two or more attributes simultaneously, and that multiplication of relations allows a child to seriate a collection of objects according to two order relations at the same time.

In general, classification (which involves equivalence relations) and seriation (which involves an asymetric relation) are at the heart of the theory of Piaget. When asked to classify, children below the age of five simply form 'figural collections'. By age seven, children can sort objects, add (union) classes, and multiply (cross-classify) classes. However, a genuine operatory classification does not exist until about age eight where the understanding of the

relative sizes of an included class to the entire class is achieved. Although ($A + A' = B$) is logically equivalent to ($A = B - A'$), many children have difficulty with the latter having mastered the former as shown by a failure to state $B > A$ (B contains more than A). The conservation of the whole (being able to hold the class B in mind when focussing on A) and the quantitative comparison of whole and part ($B > A$) are the two essential characteristics of genuine class-inclusion according to Piaget.

Recognizing that empirical research exists surrounding evidence for existence of the above groupings (i.e., replications studies) and that experiments exist which have been designed to test the theory (i.e., training studies), the present study was of a slightly different nature, being embedded in the existing psychological, mathematical, and logical theories and structures. Just how it was embedded is made clear as the study is laid out, as well as the purpose of the study. It must be emphasized that the study was not done to test the theory nor to replicate already known results, such as those produced by Smedslund (1963), Bruner and Kenney (1966), and Shantz (1967), but an employment of the theory in an applied research problem. To be sure, controversies exist concerning the validity of the theory (e.g., see Kohnstamm (1967), Braine (1959)).

Method

The theory of Piaget is a theory of development which subordinates learning to development in contrast with behavioristic theories which attempt to explain development in terms of learning (e.g., Gagne's work). As a corollary, one could view mathematical experiences (e.g., school instruction) as not being assimilated in

any genuine way in the absence of requisite cognitive structure. More specifically it would appear that work on classifications and relations would bear little fruit for children in the stage of preoperational representation. However, as Sullivan (1967) comments: "If learning should be geared to the child's present developmental level as Piaget insists, then the problem of matching the subject matter to the growing conceptual ability of the child (i.e., present cognitive structure) is a relevant consideration." [p. 19]

Instructional Unit

Classifications and relations were the broad topics about which an instructional unit of 17 lessons was constructed. The basic operations considered in the unit were ~~was~~ conjunction, disjunction, negation, and selected mathematical relations. The unit, described in detail by Johnson (unpublished dissertation), was constructed in such a way as to help children to learn to form the following: (1) classes, (2) intersections and union of classes, (3) complements of classes, and (4) relations between classes and class elements. Physical objects were employed in the learning process so that each child could be actively involved. Some free play was permitted and interaction with peers was often encouraged.

The first three lessons were designed to help the children to learn to form classes. Hula hoops and other representations of closed curves were used in these and other lessons throughout the unit to motivate the formation of various classes. Lessons IV, V, and VI concerned the intersection and the complements of the intersection of classes. The children were put in a conflict situation when they observed that an object could not belong to two disjoint hula hoops simultaneously, hence set intersection

was needed. Lessons XII, XIII, and XIV included activities relevant to the formation of the union of classes. Lessons XII, XIII, XIV, and XV contained activities designed to help children understand the relations 'more than', 'fewer than', and 'just as many as'. Other lessons in the unit involved reviews and exercises in the formation of various classes involving complementation, intersection, and union. Five basic posttests were then constructed to measure achievement and transfer.

Posttests

Connective Achievement Test. This test and the other four basic posttests, including material used, have been elsewhere recorded by Johnson (unpublished dissertation). The connective test was designed to measure the child's ability to use the logical connectives 'and', 'or', and 'not' with respect to the formation of classes. Two sets of material were used in the testing. One set consisted of sets of Dienes' Logic Blocks which were a subset of materials that had been used in the instructional unit. The other material set consisted of novel material which had not been used in the unit. The items of both parts of the 20 question test were isomorphic except for the differences in the material used. Six warm-up questions were included for each material set to insure that the children understood what the basic attributes of the objects were. One of the ten questions asked as a command was: "Put in the ring all the things that are either sticks or they are clothespins." If p and q represent statements and V, $\wedge$, and $\sim$ represent disjunction, conjunction, and negation respectively, then the following represent the types of statements asked: $p \vee q$, $p \vee \sim q$, $\sim(p \wedge q)$, $p \wedge q$,

$\sim p \wedge \sim q$, and $\sim p$.

Relation Achievement Test (RA). This 25 question test was designed to measure understanding of the relations 'more than', 'fewer than', 'just as many as', 'same shape as', and 'same color as'. For the first three relations, objects were mounted on pieces of posterboard similar to that indicated in Figure 1. Three posterboard arrangements were constructed for each of these three relations: (1) a vertical arrangement, (2) a horizontal arrangement, and (3) a circular arrangement. The set of number pairs used in the 'just as many as' questions were {(6,6), (7, 7), (8, 8)}. The set used by the 'more than', and 'fewer than' relations was {(5, 6), (6, 7), (7, 8)}. A 'more than', 'fewer than', and 'just as many as' question was asked for each item to insure that when a child said, for example: 'There are more A than B', he also knew that there were neither 'fewer A than B' nor 'just as many A as B'. For the four shape and color questions, four cards (containing two objects each) were constructed as follows: {(same shape, same color), (same shape, different color),(different shape, same color), (different shape, different color)}. For each card, the tester pointed and asked two questions: "Is this the same shape as that?" and "Is this the same color as that?". One warm-up question was incorporated to insure that each child was familiar with the process of 'matching'. The next three tests to be described are transfer tests with the exception of the intersecting ring items in the Multiplication of Classes and Relations Test.

Insert Figure 1 about here

Multiplication of Classes and Relations Test (MU). This test was constructed to measure the ability of children to use two or more criteria at once. Parts of this test were similar to the nine matrix tasks designed by Piaget which involved animal changes, pattern changes, and rotation of objects. Piaget's tasks were of either four-cell or six-cell matrices, with from five to eight choices located below the matrix. (Inhelder and Piaget, 1964) For the purpose of testing the ability of children to multiply classes and relations, six material sets spanning across each of the following three types of arrays were utilized by the investigator: (1) 3 x 3 matrices, (2) 2 x 2 matrices, and (3) ring intersection. The six material sets involved incorporating the pairing of dimensions as indicated by the following set: {(shape used in unit, color used in unit), (shape, color used in unit), (color, number), (shape, shading), (shape, size), (color, size)}. One of the ordered pairs involving a given material set was used in the construction of each of the 3 x 3 matrices. Subsets of each of the material sets were used in construction of the six 2 x 2 matrices and the six intersecting ring patterns, such that each of the unique material sets and dimension pairs was used in one and only 3 x 3, one 2 x 2, and one pair of intersecting rings. Although the intersection ring activity was not performed during the unit, it was very similar to some activities and was thus considered as an achievement measure. The matrix items were never solved in the instructional unit and hence, were viewed as transfer measures. For each of the eighteen items previously described, a strip of four response alternatives was constructed.

Class Inclusion Test (CI). This sixteen item test was included as a transfer measure because of its relevance to certain concepts included in the instructional unit. As indicated earlier, Piaget has hypothesized that training in the multiple attributes of objects will produce transfer to the class inclusion concept. This transfer transpires supposedly because class A is included in class B, then A possesses its attributes as well as those of B. Lesson XI concerned consecutive inclusions. Furthermore, when testing for the presence of the inclusion relation, usually quantification methods are used. Four lessons provided instruction with quantification. Many variables were taken into consideration in the construction of the test. With the exception of two items, the number of objects were assigned to materials in a random way with numbers coming from the set {2, 3, 4, 5} except where equality of numbers in sets was desired.

Transitivity Test (TR). A 10 question test incorporating a screening mode measured the ability of children to use the transitive property of the relations tested for in the Relations Test. Two items were used to test for the presence of the transitive property in each fo the five relations. A left to right and a right to left matching were used in the testing for the transitive property of the relations 'just as many as', 'more than', and 'fewer than'. The ordered triplets of numbers of objects used for testing for the above relations were (7, 7, 7) and (8, 8, 8), (8, 7, 6) and (9, 8, 7), and (6, 7, 8) and (7, 8, 9) respectively. This test was used as a transfer measure because the investigator was interested in the extent to which instruction on the relations hastens the development of a property normally not found to exist in children before age seven.

Sample

The subjects for the study were chosen from four kindergarten and four first grade classes located in or closely adjacent to Athens, Georgia. All of these children were administered an Otis-Lennon Mental Ability Test during March 24-April 1, 1970. A total of 99 first graders and 97 kindergarteners were tested. Two levels, Primary 1 and Elementary 1, of the Otis-Lennon Mental Abilities Test were utilized, where the Primary 1 Level is designed for pupils in the last half of the kindergarten and Elementary 1 Level is designed for pupils in the last half of the first grade. The test items sample the mental processes of classification, following directions, qualitative reasoning, comprehension of verbal concepts, and reasoning by analogy. K-R 20's for the Primary and Elementary Levels are .88 and .90 respectively. The two categorization variables, then, were cronological age and IQ. Only those children who had an IQ in the interval (80, 125) and a CA either in the interval (64, 76) or (77, 89) for kindergarten and first grade, respectively were included in the study. The children were further categorized by the two IQ intervals (80, 100), (105, 125). Children within the four categories thus defined were then randomly assigned to an experimental or control group after an ordered random sample of 80 subjects had been selected; 20 in each category. Thirty-five alternates were also selceted for a total of 115 children in the sample.

Administration of the Tests

Administration of the CA was to six subjects at a time.

Three subjects were seated adjacent to each other on one side of a table. The other three were seated facing them on the opposite side of the table. Subjects were separated by cardboard partitions so they could not see each other. Each subject was given a rope ring and some objects to classify. No objects were initially inside the rope rings. The order of test questions was randomized initially. The investigator read all the questions clearly and repeated if necessary. All subjects were given sufficient time to make their responses. The experimenters stood behind the subjects and recorded each response as being right (correct set of objects was placed in ring) or wrong (either items omitted or at least one incorrect item placed in ring). The normal testing time was approximately twenty-three minutes.

Administration of RA. For this test, the material sets were placed in a row on a low table in order from 1 to 17. Administration of questions 1-9 (matching relations) was done first with the sequence of presentation randomized individually for each subject. Also the question sequence was randomized for each question for each subject. Cards 10-17 (shape and color relations) followed with the sequence of presentation also randomized for each subject. Here again, the question sequence was randomized for each subject. The eight "same shape" questions asked of cards 10-17 composed questions 10-17 for this test and the eight "same color" questions composed questions 18-25 respectively. For each card, the resonse was scored "right" if the color and shape questions were both correct. The testor recorded the "yes" and "no" responses for each question asked. Average testing time was approximately twelve minutes.

Administration of MU. The eighteen items of this test were placed

in order on a low table similar to the method used with the RA. Each strip of four responses was centered and placed directly below the respective matrix or ring item. The sequence of presentation of the eighteen items was randomized for each subject. The tester checked on a score sheet the response pointed to on each response strip, whether it was the first, second, third, or fourth in order from left to right. Average testing time was approximately twelve minutes.

Insert Table 1 about here

For subtest CA_1 if all the proper objects were placed in the ring and nothing extra was placed there the answer was considered as correct. One point was given for correct answers and no points were given for incorrect answers. Subtest CA_2 was scored in a similar way. Since the tests were parallel, Subtest CA_3 was formed throught the consideration of the responses to the items in Subtests I and II. The subject was given credit for having a question right on Subtest III only if he had gotten each corresponding question right on both CA_1 and CA_2. In considering Subtest CA_3, one point was given for each question judged as right by the above procedure.

For the first nine material sets used in Subtest RA(considered as the first nine questions) the respective responses for each material set were judged as correct only if all three relational questions were all answered correctly. Otherwise the subject was considered to have missed that basic question. For the last eight material sets the subject was given credit for test items 10-17

only if he answered the shape questions correctly and was given credit for test items 18-25 only if he answered color correctly for cards 10-17, respectively. For the 25 basic test items one point was given for each item judged as correct and no points were given for an incorrect response.

For Subtests MU_3, MU_2, and MU_r, a correct response was when the subject pointed to the proper response in the set of four response items listed on the respective response strips. If the subject pointed to any other response item the response was categorized as wrong. One point was given for each correct response and no points were given for each wrong response.

Two methods were used in scoring the responses of the sixteen items in Subtest CI. Each of the first fourteen items were scored "correct" only if each of the last two questions asked ($A \subset B$: More A than B. More B than A?) were both answered correctly for the respective items. Questions 15 and 16 were considered correct only when the last two questions were responded to correctly for the respective items. For the sixteen basic questions each correct answer was assigned a value of one point and each wrong answer was assigned 0 points.

For each of the first six items in Subtest TR the items were considered as correct only if all five questions were all answered correctly for the respective items. For items 7, 8, 9, and 10 the answers were considered correct only if all four questions were answered correctly for all the items.

Design of Study and Method of Analysis

The basic design of the study was 'The Posttest-Only Control Group Design' presented by Campbell and Stanley (1966). This form

calls for initial randomization followed by an experimental treatment given to the experimental group. Twelve major null hypotheses were tested.

H_1: The mean vectors of the experimental and control groups are not different on the achievement measures.

H_2: The mean vectors of the experimental and control groups are not different on the transfer measures.

H_3: The mean vectors of the kindergarten and first-grade subjects are not different on the achievement measures.

H_4: The mean vectors of the kindergarten and first-grade subjects are not different on the transfer measures.

H_5: The mean vectors of the low and high IQ subjects are not different on the achievement measures.

H_6: The mean vectors of the low and high IQ subjects are not different on the transfer measures.

H_7: There is no significant interaction of IQ with Treatment on the achievement measures.

H_8: There is no significant interaction of Grade with Treatment on the achievement measures.

H_9: There is no significant interaction of Grade with IQ on the achievement measures.

H_{10}: There is no significant interaction of IQ with Treatment on the transfer measures.

H_{11}: There is no significant interaction of Grade with Treatment on the transfer measures.

H_{12}: There is no significant interaction of Grade with IQ on the transfer measures.

For each of the nine subtests composing the transfer and achievement measures, test statistics were computed. Also an item analysis was performed on all achievement and transfer test items. Two point biserial correlation coefficients, a phi coefficient, and a difficulty index were computed for each item. A point biserial correlation coefficient represents the degree of correlation existing between a dichotomous and a continuous variable. In the study, IQ measures and the total test scores formed by the composite of posttest scores, are the continuous variables. The dichotomous variables are the individual items and are scored as either right or wrong. Correlations involving IQ and total scores provide indices of validity and reliability respectively. Essentially, phi is a chi-square calculated on a two-way contingency table to test for independence of two random variables. The table was defined by experimental and control groups, and the ratio of subjects passing or failing each item to the total responses on that item.

The null hypotheses were tested with the use of Univariate Analysis of Variance (ANOVA) and Multivariate Analysis of Variance (MANOVA) procedures. Program MUDAID(Multivariate, Univariate, and Discriminant Analysis of Irregular Data) was used for the MANOVA, where the nine achievement and transfer measures were the response variables for all combinations of independent variables taken two at a time. Therefore three MANOVAs and 27 ANOVAs were calculated; one for each IQ (I) by Grade (G), IQ by Treatment (T), and Grade by Treatment. Levels of IQ were 80-100 (H) and 105-125 (L); levels of Treatment were

experimental (E) and control (C); and levels of Grade were kindergarten (K) and first grade (F).

Results

The results of the item analysis and multivariate and univariate analyses are presented in this section. All data analyzed in the item analysis section were obtained from all 111 subjects and alternates administered all the posttest measures. The multivariate and univariate analyses are limited to the 80 subjects selected for the study.

Item Analysis

A phi-coefficient was calculated for each of the 99 items. Utilizing a significant ϕ ($p<.05$), items which were discriminators between the experimental and control groups were found for each test. From the array of data in Table 2, it can easily be seen that there was only one item which discriminated in favor of the control group out of the total 99 items.

Insert Table 2 about here

Two of the subtests deserve special discussion in that all or a majority of the items of those tests were nondiscriminators. First, in the case of the RA test, the 16 items which involved usage of the relations 'same shape as' and 'same color as' were extremely easy for all subjects, and thereby were excluded from all other analyses. Secondly, four of the six items composing the MU_2 test were nondiscriminators. It appeared that much guessing was done on this test, as the average score was approximately the same as chance would allocate.

One of the four nondiscriminators on this test was excluded from all further analyses. Ten other items were also excluded from the analysis with undesirable item characteristics (very hard or very easy items with low or negative point biserial correlations with the total test or IQ). Nine of these ten items were nondiscriminators; six for the achievement measures and three for the transfer measures. Seventy-two items were retained for the analysis of variance.

Multivariate and Univariate Analyses

The necessary subtest information is tabulated in Table 3. The internal-consistency reliabilities are quite substantial indicating good homogeniety of the test items. The multivariate and univariate analyses of variance are given for the direct

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Insert Table 3 about here

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achievement measures (CA_1, CA_2, CA_3, RA, MU_r) and transfer measures (MU_3, MU_2, CI, TR) for the two classification variables (Grade and IQ) each considered in conjunction with the treatment variable, and also considered in conjunction with each other.

Analyses of Achievement Measures. For the purpose of testing the hypotheses related to achievement the five achievement subtests were considered concomitantly as response variables in the MANOVA and were considered singly in ANOVAs. In the MANOVA analysis of T vs I, the likelihood ratio test statistic $x^2 = 113.30$ was significant ($p<.01$), indicating significant differences in the mean vectors presented in Table 4. As indicated in Table 5, the main

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Insert Tables 4, 5 about here

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effects due to T and I and the intersection of T and I were significant. The test of all F-values in Table 5 is done using p and (N-3-p) degrees of freedom where p is the number of response variables and N is the number of subjects. In this analysis p is 5 and N is 80. Also, $F_{.05}(5, 72) = 2.35$ and $F_{.01}(5, 72) = 3.28$.

In order to further interpret the main effects of T, I, and T x I, five univariate analyses were performed. The results in terms of F-values for these analyses and also T vs G and I vs G are included within Table 6. It is noted that for each of the

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Insert Table 6 about here

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five response variables there existed a significant ($p<.01$) F for both T and I. This indicates that performance of E and C and also L and H were significantly different on all achievement subtests.

A significant interaction ($p<.05$) of T with I occurred only on CA_2 (involving "and", "or", and "not") and MU_r (pertaining to intersections of classes). The significant interaction indicates that on these subtests the performance of control subjects was not like the performance of experimental subjects across the two levels of IQ. Table 4 indicates that

on these subtests, the higher IQ experimental subjects performed better than any other group.

In the MANOVA analysis of T vs G, the likelihood ratio test statistic $x^2 = 71.43$ was significant ($p<.01$), indicating significant differences in the mean vectors presented in Table 7.

Insert Table 7 about here

The only main effect that was significant in this analysis, as indicated in Table 5, was T. Again, univariate analyses were performed to further interpret the main effect. As shown in Table 6, significance ($p<.01$) was achieved on each of the five subtests if and only if the effect was T. Although the effect G was not significant on any measure, it is quite noticeable from Table 7 that the first graders appeared to benefit somewhat more from instruction on the connectives "and", "or", and "not".

The final two-way analysis dealt with the factors of I and G. The likelihood ratio test statistic $x^2 = 27.41$ was significant ($p<.01$) indicating significant difference in the mean vectors persented in Table 8. As indicated in Table 5, the only main

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Insert Table 8 about here

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effect that was significant was I. Hence, for the effects of I and G, considered concomitantly, significant differences on achievement existed between the two levels of intelligence used in the study. Table 6 shows that again all F-values for the I effect were significant ($p<.01$). As can be seen from Table 8, for all five subtests the

mean scores of the high intelligence group were greater than for the low intelligence group and first graders performed better (but not significantly) than or approximately equivalent to kindergarteners. On the basis of the results listed in Tables 5 and 6, hypotheses H_1 and H_5 were rejected and H_3, H_7, H_8, and H_9 were accepted. Hence, for the achievement scores, the factors IQ and Treatment significantly affected performance. First graders performed better, but not significantly better, than kindergarteners on all achievement measures.

Analyses of Transfer Measures

The four transfer subtests were the response variables considered concomitantly in MANOVAs and separately in ANOVAs for the purpose of testing the hypotheses related to transfer effects. For the MANOVA analysis of T vs I, the likelihood ratio test statistic $x^2 = 60.19$ was significant ($p<.01$). This indicates that the mean vectors presented in Table 9 are significantly different from each other. As illustrated in Table 10, the main effects due to T and I are significant but the

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Insert Tables 9, 10 about here

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interaction of T with I was significant. The test of all F-values in Table 10 is done using p and (N-3-p) degrees of freedom as was the case with the achievement measures. However, for the transfer measures p is 4 and N is 80. For the new value of p, $F_{.05}(4, 73) = 2.49$ and $F_{.01}(4, 73) = 3.59$.

To assist the investigator in interpreting the main effects of T, I, and T x I more precisely, four univariate analyses were

performed. F-values for these analyses and also T vs G and I vs G are reported in Table 11. For MU_3 and TR significance was

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Insert Table 11 about here

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maintained ($p<.01$) for the main effect T. A significant F ($p<.05$) was computed for MU_2 but a non-significant F was computed for CI. The results were slightly different for the main effect of I. Here, significance ($p<.01$) was established for CI and TR, and for MU_2 there was significance at the .05 level. No significance was found for the main effect of I on MU_3. It is not known why the main effect of I was significant for MU_2 and not for MU_3. One possible explanation is that the subjects of greater intelligence were able to use the fewer cues available in MU_2 more proficiently than subjects of lesser intelligence. Table 9 indicates that significant differential performance always favors the experimental and high IQ groups.

For the MANOVA performed on the pair of factors T and G, the likelihood ratio test statistic $x^2 = 26.04$ was significant ($p<.01$), indicating significant differences in the mean vectors presented in Table 12. Only the main effect of T was significant ($p<.01$) as indicated in Table 10. Treatment was significant

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Insert Table 12 about here

($p<.01$) for MU_3 and TR, and was significant ($p<.05$) for MU_2, as

given in Table 11. Hence, for those three variables, performance of subjects in the two levels of T differed significantly. Table 12 reveals that for all variables for which the main effect of T was significant, Experimentals outperformed Controls.

The first two-way analysis was done with the pair of factors I and G. The likelihood ratio test statistic $x^2 = 35.48$ was significant ($p<.01$) indicating significant differences in the mean vectors presented in Table 13. As illustrated in Table 10, only the main effect of I was significant ($p<.01$). Table 11

Insert Table 13 about here

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reveals that the main effect of I was significant ($p<.01$) for CI and TR and was significant ($p<.05$) for MU_2. Hence, IQ plays an important role in performance measured by those variables. No other significant main effects were found. Table 13 indicates that responses favored the high intelligence and first-grade levels.

From the results indicated in Tables 10 and 11, hypotheses H_2 and H_5 were rejected and H_4, H_{10}, H_{11}, and H_{12} were accepted. Therefore transfer to related area was found to differ significantly depending on levels of I and T. As with the achievement measures, the more intelligent subjects performed better than the less intelligent subjects and the experimental subjects performed better than the control subjects.

Discussion

There is substantial evidence in this study that kindergarten and first-grade children can be taught (1) to form

classes based on the intersection, union, and negation of attributes of objects, and (2) to make correct "pre-number" quantitative comparisons of sets of objects. Mastery was not required, although significant differences were noted between Experimentals and Controls. Furthermore, this increase in achievement was accompanied by some transfer to related activities. The main effects of Treatment and IQ were very significant on both achievement and transfer measures but the main effect of Grade was not significant on any measure. The powerful effect of intelligence may be attributed to many causes. Perhaps, many of the components of the cognitive structure and equilibration process as described by Piaget are actually measured by an IQ test. If that is true, then possibly children of greater intelligence, as determined by such IQ measures, are at higher levels in Piaget's hierarchy initially and are "ready" for additional instruction.

At any rate, it is quite important for understanding the results of this study to distinguish between two types of experience--physical experience and logical mathematical experiences. According to Piaget (1964) physical experience "consists of acting upon objects and drawing some knowledge about the objects by abstraction from the objects." [p. 11] Piaget (1964) states further that in logical-mathematical experiences "knowledge is not drawn from the objects, but it is drawn from the actions effected on the objects." [p. 12] If a child is asked to place all the objects possessing a given attribute inside a ring, he can be shown his mistake

and it can be corrected. This type of activity is basically in the realm of physical knowledge. However, suppose that a child claims that there are more dogs than animals after he has pointed to the dogs and animals independently. It is impossible to correct his mistake in a way similar to that of the previous example. With the exception of the MU_r subtest, all the achievement measures fell in the realm of physical knowledge. Hence, the treatment was very effective for imparting physical knowledge. However, the MU_r subtest and the transfer measures must be considered when investigating the production of logical-mathematical knowledge.

Activities with intersecting rings were provided in the unit but in a format that differed from the intersecting ring test items. Although Experimentals performed significantly better than Controls on the MU_r subtest, it can be noted that neither group performed extremely well. Furthermore, Controls appeared to consider the three regions formed by the intersecting rings as nonoverlapping regions. Hence, improvement can be explained by hypotheses other than a genuine improvement in the formation of intersections. In the case of the CI subtest, the treatment did not produce significant differences. On this measure, intelligence produced the only significant effect. However, operatory classification was not achieved by either IQ group because the higher IQ group only scored about 37 per cent and the lower group only scored about 25 per cent where the expected mean based on guessing is 20 per cent. Improvement on the transitivity items can be attributed to clarity of language rather than to usage of the transitivity property. Items based on the relation of shape and color contributed greatly to

the rather high mean scores of the Transitivity subtest. Mean scores for Controls and Experimentals on matching relations were 30 and 55 per cent, respectively; while the analogous mean for the shape and color relations were 86 and 97 per cent, respectively. The matrix items provided the strongest evidence for an improvement in logical thinking, although the Genevans claim that it is difficult to distinguish between graphic and operational solutions. There was some evidence that the most substantial improvement existed for the high ability first graders.

In conclusion, the unit produced substantial improvement in physical knowledge but very little improvement in operatory classification. A structure for judging the effectiveness of the unit was provided by the cognitive development theory of Piaget. When considering the results of the study and observing the way in which addition and subtraction are presented in school mathematics curricula, a serious problem is revealed. The problem is that children are being presented with concepts they are unable to handle. In a subtraction problem such as $9 - 5 = 4$, if a child thinks that the difference is larger than the minuend he might just as well write something like $5 - 9 = 4$.

Although there was nearly a significant difference in achievement between kindergarten and first-grade children on CA_2, it is recommended that instruction similar to that used in the unit be begun at the kindergarten level because there were no significant differences in achievement between these grades on any subtest. However, more research with a more generalized population is highly recommended before final

grade-level placement is decided upon. For example, a much deeper investigation is needed concerning the actual relations that exist between the words "and", "or", and "not" and the growth of conjunction, disjunction, and negation concepts respectively. These should be investigated at various grade levels in conjunction with other concepts such as conservation of various relations as discussed by Piaget. The positive transfer made to the transitive property of the equivalence and order relation used in the unit was an interesting outcome. Various properties of the multitude of equivalence and order relations existing in the mathematics curriculum warrant similar investigations. It was noted that relations such as 'same shape as' and 'same color as' and the transitive property of these relations were very easy even for kindergarteners. Very little if any instruction is required in kindergarten for such relations.

IQ should be considered when arranging instruction based on the concepts in this study. Three of the reasons for this are as follow: (1) there was significant interaction ($p<.05$) of treatment with IQ on MU_r with the best performance by the high IQ subjects, (2) among the best discriminators between levels of intelligence was RA, and (3) the intelligence factor was significant on the transfer subtests CI and TR. This is worthy of note because these two subtests occupy key positions in the theory of Piaget. IQ was the only factor where significance was attained for CI. In such areas as those just mentioned, a thorough analysis needs to be made concerning the

relation that exists between Piaget's classification of mental operations and the degree to which these operations are measured on various IQ tests. This finding could have far-reaching implications for arranging mathematics instruction at various age levels.

At this point in time it is uncertain exactly what abilities the 3 x 3 and 2 x 2 matrix questions and the intersecting ring questions are measuring. There exists good, but inconclusive, evidence that the intersecting ring questions are measuring the same type of ability as the matrix questions. Future investigations need to incorporate other methods when investigating the intersection concept. It is assumed that the improvement in cross-classification was done through the "intersection of attribute" activities of the unit. However, it is strongly recommended that the relation existing between two attributes and a total cross-classification be investigated further. As indicated previously, Piaget has hypothesized that cross-classification, as measured by matrix activities, develops at about age seven and the intersection of simple attributes at about age nine. The present study shows that instruction in one area will perhaps hasten the development of the other operation. Any such transfer is important to education.

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FIGURE 1

SAMPLE CARDS FROM RA TEST

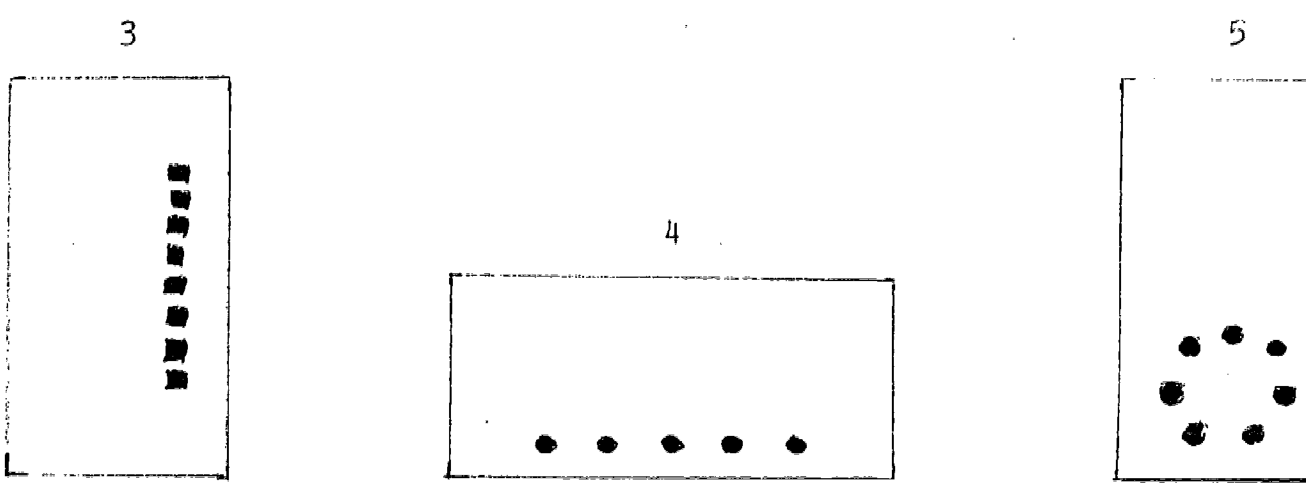

TABLE 1

FORMATION OF SUBTESTS

No. of Items	Subtests	Content of Subtest
10	CA_1	First ten questions of the CA
10	CA_2	Last ten questions of the CA(novel material
10	CA_3	Intersection of Tests I and II
25	RA	Same as the RA
6	MU_r	Last six questions of the MU (intersection rings)
6	MU_3	First six questions of the MU (3x3 matrices
6	MU_2	Second group of six questions of the MU (2x2 matrices)
16	CI	Same as the CI
10	TR	Same as the TR

TABLE 2

FREQUENCY OF ITEMS: DISCRIMINATORS AND NONDISCRIMINATORS[a]

		Discriminators		Nondiscriminators
No. of Items	Subtest	Experimental	Control	
10	CA_1	8		2
10	CA_2	7		3
10	CA_3	8		
25	RA	7	1*	15* + 2
6	MU_r	5		1
6	MU_3	4		2
6	MU_2	2		4
16	CI	0		16
10	TR	7		3

a*Items excluded from data analysis

TABLE 3

SUBTEST STATISTICS

No. of Item	Subtest	Reliability (KR-20)	Grand Mean
9	CA_1	.72	5.09
7	CA_2	.65	3.70
9	CA_3	.74	3.88
9	RA	.82	5.89
5	MU_r	.67	1.39
6	MU_3	.70	3.23
5	MU_2	.58	2.35
13	CI	.75	3.78
9	TR	.79	6.13

TABLE 4

SUBCLASS MEANS: T vs. I (ACHIEVEMENT SUBTESTS)

Subtest	Low	High	Means
Experimentals			
CA_1	5.35	7.40	6.38
CA_2	4.15	5.95	5.05
CA_3	4.25	6.35	5.30
RA	5.50	8.60	7.05
MU_r	1.55	2.85	2.20
Controls			
CA_1	3.20	4.40	3.80
CA_2	2.05	2.65	2.35
CA_3	1.80	3.10	2.45
RA	3.25	6.20	4.72
MU_r	0.55	0.60	0.58
Means			
CA_1	4.28	5.90	5.09
CA_2	3.10	4.30	3.70
CA_3	3.02	4.72	3.88
RA	4.38	7.40	5.89
MU_r	1.05	1.72	1.39

TABLE 5

F VALUES FOR MANOVA OF ACHIEVEMENT SUBTESTS[a]

Analysis	Factor	F
T vs. I	T	29.66**
	I	10.06**
	T x I	2.52*
T vs. G	T	20.32**
	G	<1
	T x G	1.13
I vs. G	I	5.43**
	G	<1
	I x G	<1

[a]* = .05 level of significance
** = .01 level of significance

TABLE 6

ANOVA F VALUES FOR ACHIEVEMENT MEASURES[a]

Type Variation	CA_1	CA_2	CA_3	RA	MU_r
T	60.22**	114.20**	80.54**	17.31**	44.97*
I	23.98**	22.56**	28.65**	29.30**	7.76*
T x I	1.64	5.64*	1.59	< 1	6.65*
T	46.05**	87.60**	59.87**	12.58**	38.84*
G	< 1	2.43	1.49	< 1	< 1
T x G	< 1	1.47	1.49	< 1	2.07
I	13.37**	8.92**	13.96**	23.90**	4.69*
G	< 1	1.25	< 1	< 1	< 1
I x G	< 1	< 1	< 1	< 1	1.09

a*= .05 level of significance
**= .01 level of significance

TABLE 7

SUBCLASS MEANS: T vs. G (ACHIEVEMENT SUBTESTS)

Subtest	Kindergarten	First Grade	Means
Experimentals			
CA_1	6.35	6.40	6.38
CA_2	5.00	5.10	5.05
CA_3	5.30	5.30	5.30
RA	6.75	7.35	7.05
MU_r	2.40	2.00	2.20
Controls			
CA_1	3.45	4.15	3.80
CA_2	1.95	2.75	2.35
CA_3	2.00	2.90	2.45
RA	4.90	4.55	4.72
MU_r	0.40	0.75	0.58
Means			
CA_1	4.90	5.28	5.09
CA_2	3.48	3.92	3.70
CA_3	3.65	4.10	3.88
RA	5.83	5.95	5.89
MU_r	1.40	1.38	1.39

TABLE 8

SUBCLASS MEANS: I vs. G (ACHIEVEMENT SUBTESTS)

Subtest	Kindergarten	First Grade	Means
Low			
CA_1	4.00	4.55	4.28
CA_2	2.80	3.40	3.10
CA_3	2.75	3.30	3.02
RA	4.40	4.35	4.38
MU_r	0.90	1.20	1.05
High			
CA_1	5.80	6.00	5.90
CA_2	4.15	4.45	4.30
CA_3	4.55	4.90	4.72
RA	7.25	7.55	7.40
MU_r	1.90	1.55	1.72
Means			
CA_1	4.90	5.28	5.09
CA_2	3.48	3.92	3.70
CA_3	3.65	4.10	3.88
RA	5.82	5.95	5.89
MU_r	1.40	1.38	1.39

TABLE 9

SUBCLASS MEANS: T vs. I (TRANSFER SUBTESTS)

Subtest	Low	High	Means
Experimentals			
MU_3	3.15	4.45	3.80
MU_2	2.10	3.40	2.75
CI	2.20	5.00	3.60
TR	5.80	8.35	7.08
Controls			
MU_3	2.60	2.70	2.65
MU_2	1.85	2.05	1.95
CI	3.10	4.80	3.95
TR	4.15	6.20	5.18
Means			
MU_3	2.88	3.58	3.22
MU_2	1.98	2.72	2.35
CI	2.65	4.90	3.78
TR	4.98	7.28	6.12

TABLE 10

F VALUES FOR MANOVA OF TRANSFER SUBTESTS[a]

Analysis	Factor	F
T vs. I	T	7.18**
	I	11.75**
	T x I	1.00
T vs. G	T	5.69**
	G	< 1
	T x G	< 1
I vs. G	I	9.68**
	G	< 1
	I x G	< 1

[a]** = significance of factors beyond the .01 level

TABLE 11

ANOVA F VALUES FOR TRANSFER MEASURES[a]

Type Variation	MU_3	MU_2	CI	TR
T	8.80**	5.59*	< 1	18.95**
I	3.26	4.91*	13.33**	27.77**
T x I	2.40	2.64	< 1	< 1
T	8.25**	5.11*	< 1	14.03**
G	< 1	< 1	< 1	< 1
T x G	< 1	< 1	1.86	< 1
I	2.88	4.45*	13.30**	22.47**
G	< 1	< 1	< 1	1.06
I x G	< 1	< 1	< 1	< 1

[a]*= .05 level of significance
**= .01 level of significance

TABLE 12

SUBCLASS MEANS: T vs. G (TRANSFER SUBTESTS)

Subtest	Kindergarten	First Grade	Means
Experimentals			
MU_3	3.60	4.00	3.80
MU_2	2.75	2.75	2.75
CI	3.45	3.75	3.60
TR	6.90	7.25	7.08
Controls			
MU_3	2.60	2.70	2.65
MU_2	1.80	2.10	1.95
CI	4.70	3.20	3.95
TR	4.85	5.50	5.18
Means			
MU_3	3.10	3.35	3.22
MU_2	2.28	2.42	2.35
CI	4.08	3.48	3.78
TR	5.88	6.38	6.12

TABLE 13

SUBCLASS MEANS: I vs. G (TRANSFER SUBTESTS)

Subtest	Kindergarten	First Grade	Means
<u>Low</u>			
MU_3	2.60	3.15	2.88
MU_2	1.95	2.00	1.98
CI	3.05	2.25	2.65
TR	4.70	5.25	4.98
<u>High</u>			
MU_3	3.60	3.55	3.58
MU_2	2.60	2.85	2.72
CI	5.10	4.70	4.90
TR	7.05	7.50	7.28
<u>Means</u>			
MU_3	3.10	3.35	3.22
MU_2	2.28	2.42	2.35
CI	4.08	3.48	3.78
TR	5.88	6.38	6.12